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Semileptonic lepton-number/flavour-violating τ decays in Majorana neutrino models

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ABSTRACT

Motivated by the recent investigation of neutrinoless τ -lepton decays by the CLEO collaboration, we perform a systematic analysis of such decays in a possible new-physics scenario with heavy Dirac/Majorana neutrinos, including heavy-neutrino nondecoupling effects, finite quark masses, and quark as well as meson mixings. We find that τ -lepton decays into an electron or muon and a pseudoscalar or vector meson can have branching ratios close to the experimental sensitivity. Numerical estimates show that the predominant decay modes of this kind are $\tau^- \rightarrow e^- \phi$, $\tau^- \rightarrow e^- \rho^0$, and $\tau^- \rightarrow e^- \pi^0$, with branching ratios of order 10^{-6} .

1 Introduction

Recently, the CLEO collaboration has reported their experimental results on 22 neutrinoless decay channels of the τ lepton that violate lepton flavour and/or lepton number [1]. Candidates for lepton-flavour/number-violating events have been found in the decays $\tau^- \rightarrow e^- \bar{K}^{*0}$, $\tau^- \rightarrow e^- \pi^+ K^-$, $\tau^- \rightarrow \mu^- \pi^- K^+$, and $\tau^- \rightarrow \mu^+ \pi^- K^-$. Such decays are strictly forbidden in the minimal Standard Model (SM) due to the fact that the light neutrinos, ν_e , ν_μ , and ν_τ , are exactly massless, so that chirality conservation implies lepton number/flavour conservation to all orders of the perturbative expansion. Since there is no fundamental theoretical reason for lepton number/flavour conservation in nature, future confirmation of the CLEO candidates may point towards physics beyond the SM and, in particular, to some modification of the lepton sector. Such possible lepton-number/flavour-violating effects can naturally be accounted for in the context of leptoquark models [2], left-right-symmetric models [3], R -parity-violating supersymmetric scenarios [4], or theories containing heavy Dirac and/or Majorana neutrinos [5,6].

In this paper, we will study the size of new-physics interactions in models with heavy Dirac and/or Majorana neutrinos. In such scenarios, decays of the τ lepton into three charged leptons, such as $\tau \rightarrow eee$, *etc.*, have been analyzed in Ref. [7]. Here, our main interest will be devoted to semileptonic decays of the τ lepton. Specifically, we will analyze decays of the type $\tau^- \rightarrow e^- \pi^0$, $\tau^- \rightarrow e^- \rho^0$, *etc.* In a previous work [8], three of the numerous decay channels of this type were considered in the framework of a theory with heavy Dirac neutrinos. We will extend that analysis by including lepton-number-violating interactions due to Majorana neutrinos, heavy-neutrino nondecoupling effects [7,9], finite-quark-mass contributions, Cabibbo-Kobayashi-Maskawa (CKM) quark mixings, and meson mixings. We will perform a complete analysis, which comprises all the ten decay channels of the type $\tau^- \rightarrow e^- M^0$, where M^0 denotes either a pseudoscalar or vector meson. The effect of a modified lepton sector on τ decays into two mesons, *i.e.*, $\tau^- \rightarrow l^\mp M_1^- M_2^\pm$, will be estimated in a separate communication [10].

In our calculations, we will adopt the conventions and the model described in Ref. [6]. In this minimal model, which is based on the SM gauge group, the neutrino sector is extended by the presence of a number (n_R) of neutral isosinglets leading to n_R heavy Majorana neutrinos (N_j), while the quark sector retains the SM structure. If the theory contains more than one neutral isosinglet, then the heavy-light neutrino mixing [11],

$$(s_L^{\nu_l})^2 \equiv 1 - \sum_{i=1}^3 |B_{l\nu_i}|^2 = \sum_{j=1}^{n_R} |B_{lN_j}|^2, \quad (1.1)$$

scales as $(m_D^\dagger (m_M^{-1})^2 m_D)_{ll}$, where m_D is the Dirac mass matrix related to the breaking of the $SU(2)_L$ gauge symmetry and m_M is a general $n_R \times n_R$ isosinglet mass matrix. Light neutrinos (ν_l) can radiatively acquire masses in compliance with experimental upper bounds [6], whereas the lepton-flavour-violating mixings $(s_L^{\nu_l})^2$ are dramatically relaxed and do not obey the traditional seesaw suppression relation, $(s_L^{\nu_l})^2 \propto m_\nu/m_N$. Then, the $(s_L^{\nu_l})^2$ may be viewed as free phenomenological parameters, which may be constrained by

a variety of low-energy data [11,12]. Throughout this paper, we will consider the following conservative upper limits for the flavour-violating mixings [12]:

$$(s_L^{\nu_e})^2, (s_L^{\nu_\mu})^2 < 0.015, \quad (s_L^{\nu_\tau})^2 < 0.050, \quad (s_L^{\nu_e})^2 (s_L^{\nu_\mu})^2 < 10^{-8}. \quad (1.2)$$

Note that these upper bounds are sensitive, to a large extent, to the degree of confidence level (CL) considered in the global analyses (*i.e.*, 95% or 99% CL) and further model-dependent assumptions [7].

In an n_G -generation model, the couplings of the charged- and neutral-current interactions are correspondingly mediated by the mixing matrices

$$B_{lj} = \sum_{k=1}^{n_G} V_{lk}^l U_{kj}^{\nu*}, \quad C_{ij} = \sum_{k=1}^{n_G} U_{ki}^\nu U_{kj}^{\nu*}, \quad (1.3)$$

where V^l and U^ν are the unitary matrices that are needed to diagonalize the charged-lepton and neutrino mass matrices, respectively. B and C satisfy a number of identities that guarantee the renormalizability of the model [6,13]. Such identities are found to be very helpful in order to reduce the number of free parameters present in such theories and, by the same token, to establish relations between B , C , and the heavy-neutrino masses. For definiteness, in our numerical calculations, we will use a model with two right-handed neutrinos. In such a scenario, we have [7]

$$B_{lN_1} = \frac{\rho^{1/4} s_L^{\nu_l}}{\sqrt{1 + \rho^{1/2}}}, \quad B_{lN_2} = i\rho^{-1/4} B_{lN_1}, \quad (1.4)$$

where $\rho = m_{N_2}^2/m_{N_1}^2$, with N_1 and N_2 being the heavy Majorana neutrinos. Furthermore, the mixings $C_{N_i N_j}$ are given by

$$\begin{aligned} C_{N_1 N_1} &= \frac{\rho^{1/2}}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2, & C_{N_2 N_2} &= \rho^{-1/2} C_{N_1 N_1}, \\ C_{N_1 N_2} &= -C_{N_2 N_1} = i\rho^{-1/4} C_{N_1 N_1}. \end{aligned} \quad (1.5)$$

Obviously, our minimal scenario only depends on m_{N_1} and m_{N_2} —or, equivalently, on m_{N_1} and ρ —, and $(s_L^{\nu_l})^2$, which are assumed to satisfy the constraints in Eq. (1.2).

The outline of this work is as follows. In Section 2, we will calculate analytically the branching ratios of the decay processes $\tau^- \rightarrow e^- M^0$. Technical details will be relegated to the Appendix. Our numerical results will be presented in Section 3. Section 4 contains our conclusions.

2 $\tau^- \rightarrow l'^- M^0$

Charge conservation forbids the lepton-number-violating decays of a τ lepton into a meson and an antilepton. For the same reason, the outgoing meson has to be neutral. The recent

CLEO experiment [1] observes an event for the decay $\tau^- \rightarrow e^- \bar{K}^{0*}$ within the signal region, which is still consistent with the estimated background due to hadron misidentification. The same experiment has considerably lowered the upper bounds on the rates of the decays with one ρ^0 or one K^{0*} in the final state.

The scattering-matrix element of $\tau^- \rightarrow l'^- M^0$ receives contributions from γ -exchange graphs, Z -boson-exchange graphs, and box graphs,

$$S(\tau^- \rightarrow l'^- M^0) = S_\gamma(\tau^- \rightarrow l'^- M^0) + S_Z(\tau^- \rightarrow l'^- M^0) + S_{Box}(\tau^- \rightarrow l'^- M^0). \quad (2.1)$$

Feynman diagrams pertinent to these decays are shown in Fig. 1. The γ and Z -boson amplitudes factorize into leptonic vertex corrections and hadronic pieces. The loop integrations are straightforward. The hadronic parts are local. Exploiting translation invariance, the phases that describe the centre-of-mass motion of M^0 may be isolated and one is left with space-time independent hadronic matrix elements. These phases assure four-momentum conservation. The γ and Z -boson amplitudes read

$$\begin{aligned} S_\gamma(\tau^- \rightarrow l'^- M^0) &= (2\pi)^4 \delta^{(4)}(p - p' - p_M) \frac{i\alpha_W^2 s_W^2}{4M_W^2} \bar{u}_{l'} \left[F_\gamma^{\tau l'} (\gamma^\mu - \frac{q^\mu \not{q}}{q^2})(1 - \gamma_5) \right. \\ &\quad \left. - G_\gamma^{\tau l'} \frac{i\sigma^{\mu\nu} q_\nu}{q^2} (m_\tau(1 + \gamma_5) + m_{l'}(1 - \gamma_5)) \right] u_\tau \\ &\quad \times \langle M^0 | \frac{2}{3} \bar{u}(0) \gamma_\mu u(0) - \frac{1}{3} \bar{d}(0) \gamma_\mu d(0) - \frac{1}{3} \bar{s}(0) \gamma_\mu s(0) | 0 \rangle, \\ S_Z(\tau^- \rightarrow l'^- M^0) &= (2\pi)^4 \delta^{(4)}(p - p' - p_M) \frac{i\alpha_W^2}{16M_W^2} F_Z^{\tau l'} \bar{u}_{l'} \gamma^\mu (1 - \gamma_5) u_\tau \\ &\quad \times \left(\langle M^0 | \bar{u}(0) \gamma_\mu \left(1 - \gamma_5 - \frac{8}{3} s_W^2 \right) u(0) | 0 \rangle \right. \\ &\quad \left. - \langle M^0 | \bar{d}(0) \gamma_\mu \left(1 - \gamma_5 - \frac{4}{3} s_W^2 \right) d(0) | 0 \rangle \right. \\ &\quad \left. - \langle M^0 | \bar{s}(0) \gamma_\mu \left(1 - \gamma_5 - \frac{4}{3} s_W^2 \right) s(0) | 0 \rangle \right), \end{aligned} \quad (2.2)$$

where p , p' , and p_M are the four-momenta of τ , l' , and M^0 , respectively, $q = p - p' = p_M$, $\alpha_W = \alpha_{\text{em}} / \sin^2 \theta_W \approx 0.0323$ is the weak fine-structure constant, and $u(x)$, $d(x)$, and $s(x)$ are quark-field operators acting on the meson states $|M^0\rangle$. In Eq. (2.2), $F_Z^{\tau l'}$, $F_\gamma^{\tau l'}$, and $G_\gamma^{\tau l'}$ are form factors, which may be found in Ref. [7].

The box diagram is more involved, as it contains a bilocal quark operator. Taking the difference, X , and the averaged sum of the space-time coordinates of the two hadronic vertices as integration variables, using translation invariance, and performing the integration over the leptonic space-time coordinates, one arrives at the following expression for the box amplitude:

$$S_{Box}(\tau^- \rightarrow l'^- M^0) = (2\pi)^4 \delta^{(4)}(p - p' - p_M) \frac{\alpha_W^2 \pi^2}{2} \sum_{i=1}^{n_G + n_R} B_{l'i} B_{\tau i}^* \int \frac{d^4 l}{(2\pi)^4}$$

$$\begin{aligned}
& \times \bar{u}_{l'} \gamma^\mu \frac{\not{p}' + \not{l}}{(p' + l)^2 - m_{n_i}^2} \gamma^\nu (1 - \gamma_5) u_\tau \frac{1}{l^2 - M_W^2} \frac{1}{(q - l)^2 - M_W^2} \\
& \times \left[\sum_{d_{a,b}=d,s} \sum_{i=1}^{n_G} V_{u_i d_b}^* V_{u_i d_a} \int d^4 X e^{-i(l-q/2)X} \right. \\
& \times \langle M^0 | \bar{d}_b \left(\frac{X}{2} \right) \gamma_\mu (1 - \gamma_5) S_F^{(u_i)}(X) \gamma_\nu (1 - \gamma_5) d_a \left(-\frac{X}{2} \right) | 0 \rangle \\
& - \sum_{i=1}^{n_G} V_{ud_i}^* V_{ud_i} \int d^4 X e^{-i(l-q/2)X} \\
& \times \langle M^0 | \bar{u} \left(-\frac{X}{2} \right) \gamma_\nu (1 - \gamma_5) S_F^{(d_i)}(-X) \gamma_\mu (1 - \gamma_5) u \left(\frac{X}{2} \right) | 0 \rangle \Big] \\
& + (W \rightarrow G), \tag{2.3}
\end{aligned}$$

where $(W \rightarrow G)$ stands for the terms obtained by replacing one or two W bosons with unphysical charged Higgs bosons and $S_F^{(u_i, d_i)}(x)$ are the u_i - and d_i -quark propagators in coordinate space. An exact expression for the quark propagator is not known, but using the free-quark propagator is known to yield a good approximation for large momentum transfers. The corresponding box amplitude which contains only free quarks, receives its dominant support from momenta with virtualities comparable to M_W (if the virtual lepton is light) or larger. Only a small fraction of the Feynman-parameter space permits potentially large contributions from low squared-momentum values. Therefore, it is plausible to approximate the quark propagator with the free-particle propagator and neglect the momenta of the external leptons and the X dependence of the quark wave functions as well. Thus, we recover the free-quark expressions for the box functions and evaluate the hadronic matrix elements by taking the quark current operators to be local. In this way, Eq. (2.3) simplifies to

$$\begin{aligned}
S_{Box}(\tau^- \rightarrow l'^- M^0) &= (2\pi)^4 \delta^{(4)}(p - p' - p_M) \frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \\
&\times \left[F_{Box}^{\tau l' uu} \langle M^0 | \bar{u}(0) \gamma^\mu (1 - \gamma_5) u(0) | 0 \rangle \right. \\
&- \sum_{d_{a,b}=d,s} F_{Box}^{\tau l' d_a d_b} \langle M^0 | \bar{d}_a(0) \gamma^\mu (1 - \gamma_5) d_b(0) | 0 \rangle \Big], \tag{2.4}
\end{aligned}$$

where $F_{Box}^{\tau l' d_a d_b}$ and $F_{Box}^{\tau l' uu}$ may be found in Appendix A.

To calculate hadronic matrix elements, we invoke the hypothesis of the partial conservation of axial vector currents (PCAC) [14,15,16],

$$A_\mu^P(x) = i\sqrt{2}f_P \partial_\mu P(x) + \dots, \tag{2.5}$$

where the dots denote terms not contributing to the meson-vacuum amplitude, f_P is the decay constant of the pseudoscalar meson P , represented by the field $P(x)$, and $A_\mu^P(x)$ is the axial-vector current having the same quark content as P . The pion decay constant is $f_\pi = 92$ MeV. Furthermore, we exploit the vector-meson dominance (VMD) relation [15,16],

$$j_{em}^\mu(x) = \frac{m_\rho^2}{2\gamma_\rho} \rho^\mu(x) + \frac{m_\omega^2}{2\sqrt{3}\gamma_\omega} \omega^\mu(x) \sin \theta_V + \frac{m_\phi^2}{2\sqrt{3}\gamma_\phi} \phi^\mu(x) \cos \theta_V, \tag{2.6}$$

and its extension for any vector current [16],

$$V_\mu^{\tilde{V}}(x) = \frac{m_{\tilde{V}}^2}{\sqrt{2}\gamma_{\tilde{V}}} \tilde{V}_\mu(x). \quad (2.7)$$

In Eq. (2.6), $j_{em}^\mu(x)$ is the electromagnetic current, $\rho^\mu(x)$, $\omega^\mu(x)$, and $\phi^\mu(x)$ are the ρ -, ω -, and ϕ -meson fields, respectively, γ_ρ , γ_ω , and γ_ϕ measure the strengths of their couplings to the photon, and θ_V is the usual mixing angle of the octet and singlet vector-meson states. In Eq. (2.7), $V_\mu^{\tilde{V}}$ is the vector field having the same quark content as the vector meson field \tilde{V} . Equation (2.7) is based on the assumption that the dominant contribution to the form factors is due to the vector mesons, which works very well for the electromagnetic current [17].

The calculation of the hadronic matrix elements proceeds as follows. One expresses the quark operators, which appear in the hadronic matrix elements, in terms of the axial-vector $[A_\mu^P(x)]$ and vector currents $[V_\mu^{\tilde{V}}(x)]$ that have the same quark content as the produced pseudoscalar (P) and vector mesons (\tilde{V}). Then, one applies Eqs. (2.5) and (2.7). The relevant matrix elements read

$$\begin{aligned} \langle 0 | A_\mu^P(x) | M(p_M) \rangle &= \delta_{MP} \sqrt{2} f_{PP\mu} e^{-ip_P x}, \\ \langle 0 | V_\mu^{\tilde{V}}(x) | M(p_M) \rangle &= \delta_{M\tilde{V}} \frac{m_{\tilde{V}}^2}{\sqrt{2}\gamma_{\tilde{V}}} \epsilon_{\tilde{V}\mu}(p_{\tilde{V}}, \lambda_{\tilde{V}}) e^{-ip_{\tilde{V}} x}, \end{aligned} \quad (2.8)$$

where $\epsilon_{\tilde{V}\mu}$ stands for the polarization vector of the vector boson \tilde{V} , and the Kronecker symbols, δ_{MP} and $\delta_{M\tilde{V}}$, assure that the matrix elements give non-zero contributions only if the final-state quantum numbers match those of the vector and axial-vector currents. The matrix elements appropriate to mesons in the final state emerge from Eq. (2.8) by Hermitean conjugation.

The decomposition of the vector and axial-vector currents into meson field operators depends on the quark content of the meson (for the pseudoscalar mesons, see Table I). The quark content of pseudoscalar mesons having zero isospin and zero hypercharge is not yet definitely established [18,19]. The mixing of SU(3)-octet and SU(3)-singlet meson states with zero isospin and zero hypercharge is usually parameterized by some angle, θ_P , which is not precisely known. The corresponding mixing angle for vector mesons is called θ_V . From the study of ϕ decays it is known that θ_V is very close to the ideal value $\arctan(1/\sqrt{2})$. Notice that the state $|M\rangle$ and the corresponding field $M(x)$ have opposite quantum numbers. This is due to the convention $\langle 0 | M(x) | M(p) \rangle = e^{-ipx} \epsilon_M(p)$, *i.e.*, the meson field annihilates the corresponding meson state. In the second line of Table I, we indicate the relevant creation and annihilation operators that are contained in the meson states. Here, b_u and d_s are the annihilation operators of the quark u and the antiquark \bar{s} , respectively, and b_u^\dagger and d_s^\dagger are their creation operators. The quark structure of the vector-meson states and fields may be read off from Table I after the replacements $K^\pm \rightarrow K^{\pm 0}$, $\pi^\pm \rightarrow \rho^\pm$, $\pi^0 \rightarrow \rho^0$, $K^0 \rightarrow K^{0*}$, $\bar{K}^0 \rightarrow \bar{K}^{0*}$, $\eta_{8,1} \rightarrow \phi_{8,1}$, $\eta \rightarrow \phi$, $\eta' \rightarrow \omega$, and $\theta_P \rightarrow \theta_V$.

Following the procedure outlined above, we obtain the following expressions for the $\tau^- \rightarrow l'^- M^0$ matrix elements:

$$\begin{aligned}
T(\tau^- \rightarrow l'^- K^0) &= \frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \sqrt{2} f_K p_K^\mu F_{Box}^{\tau l' sd}, \\
T(\tau^- \rightarrow l'^- \bar{K}^0) &= -\frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \sqrt{2} f_K p_K^\mu F_{Box}^{\tau l' ds}, \\
T(\tau^- \rightarrow l'^- \pi^0) &= -\frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau f_\pi p_\pi^\mu [2F_Z^{\tau l'} + F_{Box}^{\tau l' uu} + F_{Box}^{\tau l' dd}], \\
T(\tau^- \rightarrow l'^- \eta) &= \frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau f_\eta p_\eta^\mu \left[-\left(\frac{2c_P}{\sqrt{3}} + \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_Z^{\tau l'} \right. \\
&\quad \left. + \left(-\frac{c_P}{\sqrt{3}} + \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' uu} + \left(\frac{c_P}{\sqrt{3}} - \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' dd} \right. \\
&\quad \left. - \left(\frac{2c_P}{\sqrt{3}} + \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' ss} \right], \\
T(\tau^- \rightarrow l'^- \eta') &= \frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau f_{\eta'} p_{\eta'}^\mu \left[\left(\frac{\sqrt{2}c_P}{\sqrt{3}} - \frac{2s_P}{\sqrt{3}}\right) F_Z^{\tau l'} \right. \\
&\quad \left. - \left(\frac{\sqrt{2}c_P}{\sqrt{3}} + \frac{s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' uu} + \left(\frac{\sqrt{2}c_P}{\sqrt{3}} + \frac{s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' dd} \right. \\
&\quad \left. + \left(\frac{\sqrt{2}c_P}{\sqrt{3}} - \frac{2s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' ss} \right], \\
T(\tau^- \rightarrow l'^- K^{0*}) &= -\frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \frac{m_{K^{0*}}^2}{\sqrt{2}\gamma_{K^{0*}}} \epsilon_{K^{0*}}^\mu F_{Box}^{\tau l' sd}, \\
T(\tau^- \rightarrow l'^- \bar{K}^{0*}) &= \frac{i\alpha_W^2}{16M_W^2} \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \frac{m_{K^{0*}}^2}{\sqrt{2}\gamma_{K^{0*}}} \epsilon_{K^{0*}}^\mu F_{Box}^{\tau l' ds}, \\
T(\tau^- \rightarrow l'^- \rho^0) &= \frac{i\alpha_W^2}{16M_W^2} \frac{m_{\rho^0}^2}{\gamma_{\rho^0}} \epsilon_{\rho^0}^\mu \left\{ 2s_W^2 \bar{u}_{l'} \left[F_\gamma^{\tau l'} \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) (1 - \gamma_5) \right. \right. \\
&\quad \left. \left. - G_\gamma^{\tau l'} \frac{i\sigma_{\mu\nu} q^\nu}{q^2} (m_\tau(1 + \gamma_5) + m_{l'}(1 - \gamma_5)) \right] u_\tau \right. \\
&\quad \left. + \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \left[c_{2W} F_Z^{\tau l'} + \frac{1}{2} F_{Box}^{\tau l' uu} + \frac{1}{2} F_{Box}^{\tau l' dd} \right] \right\}, \\
T(\tau^- \rightarrow l'^- \phi) &= \frac{i\alpha_W^2}{16M_W^2} \frac{m_\phi^2}{\gamma_\phi} \epsilon_\phi^\mu \left\{ \frac{2s_W^2 c_V}{\sqrt{3}} \bar{u}_{l'} \left[F_\gamma^{\tau l'} \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) (1 - \gamma_5) \right. \right. \\
&\quad \left. \left. - G_\gamma^{\tau l'} \frac{i\sigma_{\mu\nu} q^\nu}{q^2} (m_\tau(1 + \gamma_5) + m_{l'}(1 - \gamma_5)) \right] u_\tau \right. \\
&\quad \left. + \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \left[\left(\frac{c_V}{\sqrt{3}} c_{2W} + \frac{s_V}{\sqrt{6}}\right) F_Z^{\tau l'} + \left(\frac{c_V}{2\sqrt{3}} - \frac{s_V}{\sqrt{6}}\right) F_{Box}^{\tau l' uu} \right. \right. \\
&\quad \left. \left. - \left(\frac{c_V}{2\sqrt{3}} - \frac{s_V}{\sqrt{6}}\right) F_{Box}^{\tau l' dd} + \left(\frac{c_V}{\sqrt{3}} + \frac{s_V}{\sqrt{6}}\right) F_{Box}^{\tau l' ss} \right] \right\}, \\
T(\tau^- \rightarrow l'^- \omega) &= \frac{i\alpha_W^2}{16M_W^2} \frac{m_\omega^2}{\gamma_\omega} \epsilon_\omega^\mu \left\{ \frac{2s_W^2 s_V}{\sqrt{3}} \bar{u}_{l'} \left[F_\gamma^{\tau l'} \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) (1 - \gamma_5) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -G_\gamma^{\tau l'} \frac{i\sigma_{\mu\nu} q^\nu}{q^2} \left(m_\tau(1 + \gamma_5) + m_{l'}(1 - \gamma_5) \right) \Big] u_\tau \\
& + \bar{u}_{l'} \gamma_\mu (1 - \gamma_5) u_\tau \left[\left(\frac{s_V}{\sqrt{3}} c_{2W} - \frac{c_V}{\sqrt{6}} \right) F_Z^{\tau l'} + \left(\frac{s_V}{2\sqrt{3}} + \frac{c_V}{\sqrt{6}} \right) F_{Box}^{\tau l' uu} \right. \\
& \left. - \left(\frac{s_V}{2\sqrt{3}} + \frac{c_V}{\sqrt{6}} \right) F_{Box}^{\tau l' dd} + \left(\frac{s_V}{\sqrt{3}} - \frac{c_V}{\sqrt{6}} \right) F_{Box}^{\tau l' ss} \right] \Big\}, \tag{2.9}
\end{aligned}$$

where we have introduced the short-hand notations $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, $c_{2W} = \cos 2\theta_W$, and similarly for θ_P and θ_V .

The branching ratios for pseudoscalar mesons can be compactly written in the form

$$B(\tau^- \rightarrow l'^- M^0) = \frac{1}{8\pi} \frac{m_\tau}{\Gamma_\tau} \frac{\lambda^{\frac{1}{2}}(m_\tau^2, m_{l'}^2, m_{M^0}^2)}{m_\tau^2} |a_{M^0}|^2 \frac{(m_\tau^2 - m_{l'}^2)^2 - m_{M^0}^2(m_\tau^2 + m_{l'}^2)}{m_\tau^2}, \tag{2.10}$$

where the form factors a_{M^0} are listed in Appendix A, $\Gamma_\tau = 2.16 \cdot 10^{-12}$ GeV is the total width of the τ lepton measured experimentally, and $\lambda(x, y, z) = (x - y - z)^2 - 4yz$. Similarly, in the case of the vector mesons, one finds

$$\begin{aligned}
B(\tau^- \rightarrow l'^- M^0) = & \frac{1}{8\pi} \frac{m_\tau}{\Gamma_\tau} \frac{\lambda^{\frac{1}{2}}(m_\tau^2, m_{l'}^2, m_{M^0}^2)}{m_\tau^2} \left[|c_{M^0}|^2 \left\{ -\frac{12m_{l'}^2}{m_{M^0}^2} \right. \right. \\
& + \frac{(m_\tau^2 + m_{l'}^2)(2(m_\tau^2 - m_{l'}^2)^2 - m_{M^0}^2(m_\tau^2 + m_{l'}^2) - m_{M^0}^4)}{m_\tau^2 m_{M^0}^4} \Big\} \\
& + |a_{M^0} + b_{M^0}|^2 \frac{(m_\tau^2 - m_{l'}^2)^2 + m_{M^0}^2(m_\tau^2 + m_{l'}^2) - 2m_{M^0}^4}{m_\tau^2 m_{M^0}^2} \\
& \left. - 6\Re\{(a_{M^0} + b_{M^0})c_{M^0}^*\} \frac{(m_\tau^2 - m_{l'}^2)^2 - m_{M^0}^2(m_\tau^2 + m_{l'}^2)}{m_\tau^2 m_{M^0}^2} \right], \tag{2.11}
\end{aligned}$$

where a_{M^0} , b_{M^0} , and c_{M^0} may also be found in Appendix A.

3 Numerical results

In our numerical analysis, we will assume that the SM is extended by two right-handed neutrinos, as described in the Introduction. The additional parameters in this scenario are the two heavy-neutrino masses, m_{N_1} and m_{N_2} , and the three mixing angles, $s_L^{\nu_e}$, $s_L^{\nu_\mu}$, and $s_L^{\nu_\tau}$. These are free parameters of the model, which may be limited by experiment. The upper bounds on $s_L^{\nu_e}$, $s_L^{\nu_\mu}$, and $s_L^{\nu_\tau}$ are given in Eq. (1.2). On the other hand, the perturbative unitarity relations,

$$\frac{\Gamma_{N_i}}{m_{N_i}} < \frac{1}{2}, \tag{3.1}$$

lead to a global upper bound on m_{N_1} ,

$$m_{N_1}^2 \leq \frac{2M_W^2}{\alpha_W} \frac{1 + \rho^{-\frac{1}{2}}}{\rho^{\frac{1}{2}}} \left[\sum_i (s_L^{\nu_i})^2 \right]^{-1}, \tag{3.2}$$

where ρ is defined after Eq. (1.4) and it is understood that $\rho \geq 1$. In this context, we should mention that, adapting the results of Ref. [20] based on a renormalization-group analysis in a four-generation Majoron model, one may find a bound which is slightly more restrictive than Eq. (3.2) but still lies in the same ball park.

Furthermore, our results depend on hadronic observables and quark-level parameters such as the CKM-matrix elements, the quark and meson masses, the mixing angles of the meson singlet and octet states, the pseudoscalar-meson decay constants, and the coupling strengths of the vector mesons to the gauge bosons. In our calculations, we use the maximum experimental values for the CKM-matrix elements [18] and the quark-mass values [18,21]

$$\begin{aligned} m_u &= 0.005 \text{ GeV}, & m_d &= 0.010 \text{ GeV}, & m_s &= 0.199 \text{ GeV}, \\ m_c &= 1.35 \text{ GeV}, & m_b &= 4.3 \text{ GeV}, & m_t &= 176 \text{ GeV}. \end{aligned} \quad (3.3)$$

We keep all quark masses finite, since, *e.g.*, the *c*-quark and *t*-quark contributions to the box amplitudes turn out to be comparable. The mixing angle for vector-meson nonet states may be determined from the quadratic Gell-Mann–Okubo mass formula to be $\theta_V = 41.3^\circ$. We treat θ_P as a free parameter because its value is not yet well established [19,22]. For the most part, we use $\theta_P = -23^\circ$, the value extracted from $e^+e^- \rightarrow e^+e^-\gamma\gamma^* \rightarrow e^+e^-(P \rightarrow \gamma\gamma)$ experiments [22]. This value is consistent with a previous analysis [19]. For the pseudoscalar-meson decay constants, we use the experimental values [18,22],

$$\begin{aligned} f_{\pi^\pm} &= 92.4 \text{ MeV}, & f_{K^\pm} &= 113 \text{ MeV}, \\ f_{\pi^0} &= 84.1 \text{ MeV}, & f_\eta &= 94 \text{ MeV}, & f_{\eta'} &= 89.1 \text{ MeV}, \end{aligned} \quad (3.4)$$

and exploit SU(3) flavour symmetry,

$$f_{K^0} = f_{\bar{K}^0} \approx f_{K^\pm}. \quad (3.5)$$

The constants $\gamma_{\tilde{V}}$ are partly extracted from the $\tilde{V} \rightarrow e^+e^-$ decay rates, with the result that

$$\gamma_{\rho^0} = 2.519, \quad \gamma_\omega = 2.841, \quad \gamma_\phi = 3.037, \quad (3.6)$$

and partly estimated assuming SU(3) symmetry: we put $\gamma_{K^{*0}} = \gamma_{\rho^0}$ because K^{*0} and ρ^0 are members of the same SU(3) octet, while ϕ and ω are mixtures of octet and singlet states. Notice that all $\gamma_{\tilde{V}}$ values in Eq. (3.6) are very similar in size.

Having specified our input parameters, we will now discuss our numerical results. The widths for the decays with K^0 , \bar{K}^0 , K^{*0} , or \bar{K}^{*0} in the final state only receive contributions from box diagrams. The branching ratios for these decays are found to be always smaller than 10^{-14} , that is, much smaller than present experimental sensitivities ($\sim 10^{-6}$), rendering these decay modes uninteresting from the experimental point of view. Therefore, we will not pursue their study any further.

For definiteness, we will consider decays of the form $\tau^- \rightarrow e^- M^0$ only—we set $(s_L^\nu)^2 \approx 0$ to satisfy the third inequality in Eq. (1.2). Of course, our estimates are also valid for

the $\tau^- \rightarrow \mu^- M^0$ decays with $(s_L^{\nu_e})^2 = 0$. Our results for the branching ratios $B(\tau^- \rightarrow e^- \pi^0 / \eta / \eta' / \rho^0 / \phi / \omega)$ are illustrated in Figs. 2–6. Each figure describes the dependence of the branching ratios on two of the free parameters, one varied continuously and the other one in a discrete manner. All other parameters are kept fixed. Figure 2 shows the dependence of the branching ratios on $m_N = m_{N_1} = m_{N_2}$ and $(s_L^{\nu_\tau})^2$. The most promising modes are $\tau^- \rightarrow e^- \phi$, $\tau^- \rightarrow e^- \rho^0$, and $\tau^- \rightarrow e^- \pi^0$, which, for maximum values of m_N and $(s_L^{\nu_\tau})^2$, reach branching fractions

$$\begin{aligned} B(\tau^- \rightarrow e^- \phi) &\lesssim 1.6 \cdot 10^{-6}, \\ B(\tau^- \rightarrow e^- \rho^0) &\lesssim 0.9 \cdot 10^{-6}, \\ B(\tau^- \rightarrow e^- \pi^0) &\lesssim 1.0 \cdot 10^{-6}. \end{aligned} \quad (3.7)$$

This has to be compared with the present experimental bounds [1,23]

$$\begin{aligned} B(\tau^- \rightarrow e^- \rho^0) &< 4.2 \cdot 10^{-6}, \\ B(\tau^- \rightarrow e^- \pi^0) &< 1.4 \cdot 10^{-4}, \\ B(\tau^- \rightarrow \mu^- \pi^0) &< 4.4 \cdot 10^{-5}, \end{aligned} \quad (3.8)$$

at the 90 % CL. Unfortunately, $B(\tau^- \rightarrow e^- \phi)$ has not been measured yet. We conclude that an experimental investigation of $\tau^- \rightarrow e^- \phi$ and a more precise determination of $B(\tau^- \rightarrow e^- \pi^0)$ and $B(\tau^- \rightarrow \mu^- \pi^0)$ would be highly desirable. In the high- m_{N_1} limit, Fig. 2 shows the quadratic m_N dependence for all branching ratios, except for $B(\tau^- \rightarrow e^- \omega)$. In the 't Hooft–Feynman gauge, this behaviour originates mainly from the Z -boson amplitudes, $F_Z^{\tau l'}$.

At this stage, some important comments are in order. For fixed $(s_L^{\nu_i})^2$ values, N_1 and N_2 do not decouple from our theory as their masses become large as compared to M_W [7,9]. As has been mentioned in the Introduction, $s_L^{\nu_i} \propto m_D/m_M \propto m_D/m_{N_i}$, and this nondecoupling feature can be traced to the large $SU(2)_L$ Dirac components, m_D , present in our model [9]. Obviously, if we fix m_D and take the limit $m_N \rightarrow \infty$, the heavy neutrinos will decouple from our low-energy processes, leading to vanishing effects [24]. This will be illustrated in greater detail in Figs. 4 and 5. However, for heavy neutrinos, with masses in the 1–10 TeV range, there will be an interesting nondecoupling “window” arising from potentially large Dirac mass terms m_D . It is precisely this nondecoupling “window” which we are exploiting here to make our effects sizeable.

In the case of $B(\tau^- \rightarrow e^- \pi^0 / \eta / K^0)$, we recover the expressions of Ref. [8] for the Dirac-neutrino scenario if we omit the nondecoupling terms proportional to m_N^2 in Eq. (A.2) and in the Z -boson-mediated amplitudes. The results of Ref. [8] are comparable to ours for m_N of order M_W , but they fall short of our results by up to factor of 50 for m_N in the TeV region. In the case of ω production, there is a destructive effect between logarithmic and quadratic m_N -dependent nondecoupling terms coming from photon and Z -boson-mediated amplitudes, respectively, and meson-mixing effects, which show up as a minimum of the branching ratio for $m_{N_1} \approx 1.6$ TeV.

We now turn to genuine Majorana-neutrino quantum effects. Figure 3 displays the dependence of the branching fractions on the ratio m_{N_2}/m_{N_1} for the fixed values $m_{N_1} = 1$ TeV

and 0.5 TeV. We emphasize that, just like in the lepton case [7], $B(\tau^- \rightarrow e^- \pi^0 / \eta / \eta' / \rho^0 / \phi)$ assume their maximum values for $m_{N_2}/m_{N_1} = 2\text{--}4$ rather than in the Dirac scenario, $m_{N_1} = m_{N_2}$. The only exception is the decay $\tau^- \rightarrow e^- \omega$, where the maximum value is shifted to larger values of m_{N_2}/m_{N_1} , of order 20, due to the accidental cancellations mentioned above.

Figures 4 and 5 illustrate the dependence of the branching ratios on $(s_L^{\nu_\tau})^2$ and $(s_L^{\nu_e})^2$ in the heavy-Dirac-neutrino scenario with $m_{N_1} = m_{N_2} = 4$ TeV. As may be seen in Fig. 4, the $(s_L^{\nu_\tau})^2$ dependence of $B(\tau^- \rightarrow e^- \pi^0 / \eta / \eta' / \rho^0 / \phi / \omega)$ is quadratic over the most part of the $(s_L^{\nu_\tau})^2$ range and for any of the $(s_L^{\nu_e})^2$ values considered. From Fig. 5 we see that the $(s_L^{\nu_e})^2$ dependence of $B(\tau^- \rightarrow e^- \pi^0 / \eta / \eta' / \rho^0 / \phi / \omega)$ is approximately linear for $(s_L^{\nu_e})^2 < (s_L^{\nu_\tau})^2$, while it becomes quadratic for $(s_L^{\nu_e})^2 > (s_L^{\nu_\tau})^2$. The $(s_L^{\nu_\tau})^2$ and $(s_L^{\nu_e})^2$ dependences studied above are closely related to the decoupling behaviour of the isosinglet scale m_M . As we have emphasized above, in the limit $m_N \rightarrow \infty$ for constant m_D —or, equivalently, for constant m_N and vanishing m_D , *i.e.*, for $(s_L^{\nu_i})^2 \rightarrow 0$ —we should recover the decoupling limit, where the branching ratios vanish as the isosinglet mass terms m_M are sent to infinity [24]. It is then evident that the aforementioned (non)decoupling “window” is directly related to the $SU(2)_L$ Dirac terms m_D [9] and is reflected in the actual $(s_L^{\nu_\tau})^2$ and $(s_L^{\nu_e})^2$ dependences seen in Figs. 4 and 5.

In Fig. 6, we plot $B(\tau^- \rightarrow e^- \eta / \eta')$ versus $m_N = m_{N_1} = m_{N_2}$, assuming in turn the unmixed case ($\theta_P = 0$) and $\theta_P = -10^\circ$. We see that, for θ_P decreasing, $B(\tau^- \rightarrow e^- \eta')$ increases considerably, while $B(\tau^- \rightarrow e^- \eta)$ grows just slightly. This illustrates that it is important to allow for nonvanishing θ_P in realistic calculations. It is also interesting to observe that, if $\tan \theta_{\tilde{V}} = 1/(\sqrt{2}c_{2W})$, the dominant nondecoupling terms proportional to m_N^2 are quenched in $B(\tau^- \rightarrow l^- \omega)$.

4 Conclusions

Motivated by the recent experimental search for lepton-number/flavour-violating semileptonic τ -lepton decays [1], which are strictly prohibited in the SM, we have explored the potential of extensions of the SM by heavy Dirac and/or Majorana neutrinos to account for $\tau^- \rightarrow l'^- M^0$ decays, where $l' = e, \mu$ and M^0 is a neutral pseudoscalar or vector meson, with branching ratios which are in line with the experimental results. Since such models predict appreciable branching fractions for lepton-flavour/number-violating leptonic decays of the τ lepton [7], they are also expected to be promising candidates for explaining the problem at hand. In fact, we have found branching fractions in excess of 10^{-6} for the channels $\tau^- \rightarrow e^- \phi$, $\tau^- \rightarrow e^- \rho^0$, and $\tau^- \rightarrow e^- \pi^0$. Our value for $B(\tau^- \rightarrow e^- \rho)$ is comparable to the present experimental sensitivity [1]. Unfortunately, the experimental upper limit on $B(\tau^- \rightarrow e^- \pi^0)$ still exceeds our result by two orders of magnitude [23]. For some reason,

the decays $\tau^- \rightarrow e^- \phi$ or $\tau^- \rightarrow \mu^- \phi$, which prevail in our numerical estimates, have not yet been studied experimentally. At this point, we should like to encourage our experimental colleagues to undertake a search for this decay channel.

An important feature of our model is that the $\tau^- \rightarrow l'^- M^0$ decay amplitudes exhibit a quadratic dependence on the heavy-neutrino masses, m_{N_1} and m_{N_2} . This nondecoupling dependence is closely related to the large $SU(2)_L$ -breaking Dirac terms m_D that are allowed to be present in our minimal *three*-generation seesaw-type scenario [7,9]. These $m_{N_i}^2$ terms are negligible for neutrino masses below 200 GeV, but they are dominant in the TeV region, where they may lead to an enhancement by a factor of 50 of the respective analysis with these terms omitted [8]. The same nondecoupling terms give rise to a m_{N_2}/m_{N_1} dependence of the $\tau^- \rightarrow l'^- M^0$ decay amplitudes which is similar to the one encountered for the decays $\tau^- \rightarrow e^+ e^- e^-$, *etc.* [7]. In particular, semileptonic branching ratios take their maximum values for $m_{N_2}/m_{N_1} \approx 2-4$. The $\tau^- \rightarrow l'^- \omega$ decay rate is unobservably small due to a destructive meson-mixing effect, which considerably screens the dominant Z -exchange interaction.

The extension of the vector-meson dominance hypothesis to general vector currents has enabled us to calculate the decays with vector mesons in the final state. The quark content of meson wave functions, which, for instance, is reflected in the mixing angles, θ_P and $\theta_{\tilde{V}}$, is also important. We have illustrated this for the production of η , η' , and ϕ mesons.

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A Appendix

The amplitudes a_{M^0} , b_{M^0} , and c_{M^0} appearing in Eqs. (2.10) and (2.11) may be decomposed into the form factors $F_{Box}^{\tau l' d_a d_b}$, $F_{Box}^{\tau l' uu}$, $F_Z^{\tau l'}$, $F_\gamma^{\tau l'}$, and $G_\gamma^{\tau l'}$ in the following way:

$$a_{K^0}^{\tau l'} = \frac{i\alpha_W^2}{16 M_W^2} \sqrt{2} f_{K^0} F_{Box}^{\tau l' sd},$$

$$\begin{aligned}
a_{\bar{K}^0}^{\tau l'} &= -\frac{i\alpha_W^2}{16 M_W^2} \sqrt{2} f_{K^0} F_{Box}^{\tau l' ds}, \\
a_{\pi^0}^{\tau l'} &= -\frac{i\alpha_W^2}{16 M_W^2} f_{\pi^0} [2F_Z^{\tau l'} + F_{Box}^{\tau l' dd} + F_{Box}^{\tau l' uu}], \\
a_{\eta}^{\tau l'} &= \frac{i\alpha_W^2}{16 M_W^2} f_{\eta} \left[-\left(\frac{2c_P}{\sqrt{3}} + \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_Z^{\tau l'} - \left(\frac{c_P}{\sqrt{3}} - \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' uu} \right. \\
&\quad \left. + \left(\frac{c_P}{\sqrt{3}} - \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' dd} - \left(\frac{2c_P}{\sqrt{3}} + \frac{\sqrt{2}s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' ss} \right], \\
a_{\eta'}^{\tau l'} &= \frac{i\alpha_W^2}{16 M_W^2} f_{\eta'} \left[\left(\frac{\sqrt{2}c_P}{\sqrt{3}} - \frac{2s_P}{\sqrt{3}}\right) F_Z^{\tau l'} - \left(\frac{s_P}{\sqrt{3}} + \frac{\sqrt{2}c_P}{\sqrt{3}}\right) F_{Box}^{\tau l' uu} \right. \\
&\quad \left. + \left(\frac{s_P}{\sqrt{3}} + \frac{\sqrt{2}c_P}{\sqrt{3}}\right) F_{Box}^{\tau l' dd} + \left(\frac{\sqrt{2}c_P}{\sqrt{3}} - \frac{2s_P}{\sqrt{3}}\right) F_{Box}^{\tau l' ss} \right], \\
a_{K^{0*}}^{\tau l'} &= -\frac{i\alpha_W^2}{16 M_W^2} \frac{m_{K^{0*}}^2}{\sqrt{2}\gamma_{K^{0*}}} F_{Box}^{\tau l' sd}, \\
a_{\bar{K}^{0*}}^{\tau l'} &= \frac{i\alpha_W^2}{16 M_W^2} \frac{m_{K^{0*}}^2}{\sqrt{2}\gamma_{K^{0*}}} F_{Box}^{\tau l' ds}, \\
a_{\rho^0}^{\tau l'} &= \frac{i\alpha_W^2}{16 M_W^2} \frac{m_{\rho^0}^2}{\gamma_{\rho^0}} [c_{2W} F_Z^{\tau l'} + \frac{1}{2} F_{Box}^{\tau l' uu} + \frac{1}{2} F_{Box}^{\tau l' dd}], \\
a_{\phi}^{\tau l'} &= \frac{i\alpha_W^2}{16 M_W^2} \frac{m_{\phi}^2}{\gamma_{\phi}} \left[\left(\frac{c_V}{\sqrt{3}} c_{2W} + \frac{s_V}{\sqrt{6}}\right) F_Z^{\tau l'} + \left(\frac{c_V}{2\sqrt{3}} - \frac{s_V}{\sqrt{6}}\right) F_{Box}^{\tau l' uu} \right. \\
&\quad \left. - \left(\frac{c_V}{2\sqrt{3}} - \frac{s_V}{\sqrt{6}}\right) F_{Box}^{\tau l' dd} + \left(\frac{c_V}{\sqrt{3}} + \frac{s_V}{\sqrt{6}}\right) F_{Box}^{\tau l' ss} \right], \\
a_{\omega}^{\tau l'} &= \frac{i\alpha_W^2}{16 M_W^2} \frac{m_{\omega}^2}{\gamma_{\omega}} \left[\left(\frac{s_V}{\sqrt{3}} c_{2W} - \frac{c_V}{\sqrt{6}}\right) F_Z^{\tau l'} + \left(\frac{s_V}{2\sqrt{3}} + \frac{c_V}{\sqrt{6}}\right) F_{Box}^{\tau l' uu} \right. \\
&\quad \left. - \left(\frac{s_V}{2\sqrt{3}} + \frac{c_V}{\sqrt{6}}\right) F_{Box}^{\tau l' dd} + \left(\frac{s_V}{\sqrt{3}} - \frac{c_V}{\sqrt{6}}\right) F_{Box}^{\tau l' ss} \right], \\
b_{K^{0*}}^{\tau l'} &= b_{\bar{K}^{0*}}^{\tau l'} = 0, \\
b_{\rho^0}^{\tau l'} &= \frac{i\alpha_W^2 s_W^2}{4 M_W^2} \frac{m_{\rho^0}^2}{2\gamma_{\rho^0}} F_{\gamma}^{\tau l'}, \\
b_{\phi}^{\tau l'} &= \frac{i\alpha_W^2 s_W^2}{4 M_W^2} \frac{m_{\phi}^2}{\gamma_{\phi}} \frac{c_V}{2\sqrt{3}} F_{\gamma}^{\tau l'}, \\
b_{\omega}^{\tau l'} &= \frac{i\alpha_W^2 s_W^2}{4 M_W^2} \frac{m_{\omega}^2}{\gamma_{\omega}} \frac{s_V}{2\sqrt{3}} F_{\gamma}^{\tau l'}, \\
c_{K^{0*}}^{\tau l'} &= c_{\bar{K}^{0*}}^{\tau l'} = 0, \\
c_{\rho^0}^{\tau l'} &= -\frac{i\alpha_W^2 s_W^2}{4 M_W^2} \frac{m_{\rho}^2}{2\gamma_{\rho}} G_{\gamma}^{\tau l'}, \\
c_{\phi}^{\tau l'} &= -\frac{i\alpha_W^2 s_W^2}{4 M_W^2} \frac{m_{\phi}^2}{\gamma_{\phi}} \frac{c_V}{2\sqrt{3}} G_{\gamma}^{\tau l'},
\end{aligned}$$

$$c_{\omega}^{\tau l'} = -\frac{i\alpha_W^2 s_W^2}{4M_W^2} \frac{m_{\omega}^2}{\gamma_{\omega}} \frac{s_V}{2\sqrt{3}} G_{\gamma}^{\tau l'}. \quad (\text{A.1})$$

The form factors $F_{Box}^{\tau l' d_a d_b}$, $F_{Box}^{\tau l' uu}$, $F_Z^{\tau l'}$, $F_{\gamma}^{\tau l'}$, and $G_{\gamma}^{\tau l'}$, which also appear explicitly in Section 2, may in turn be decomposed into elementary vertex and box functions, F_{γ} , G_{γ} , F_Z , F_{Box} , and H_{Box} . The form factors $F_Z^{\tau l'}$, $F_{\gamma}^{\tau l'}$, and $G_{\gamma}^{\tau l'}$ together with the elementary loop functions F_{γ} , G_{γ} , F_Z , and F_{Box} may be found in Ref. [7]. Here, we list $F_{Box}^{\tau l' d_a d_b}$ and $F_{Box}^{\tau l' uu}$,

$$\begin{aligned} F_{Box}^{\tau l' uu} &= \sum_{i=1}^{n_R} \sum_{j=1}^{n_G} B_{\tau N_i}^* B_{l' N_i} V_{ud_j}^* V_{ud_j} \left[H_{Box}(\lambda_{N_i}, \lambda_{d_j}) \right. \\ &\quad \left. - H_{Box}(\lambda_{N_i}, 0) - H_{Box}(0, \lambda_{d_j}) + H_{Box}(0, 0) \right] \\ &\quad + \sum_{i=1}^{n_R} B_{\tau N_i}^* B_{l' N_i} [H_{Box}(\lambda_{N_i}, 0) - H_{Box}(0, 0)], \\ F_{Box}^{\tau l' d_a d_b} &= \sum_{i=1}^{n_R} \sum_{j=1}^{n_G} B_{\tau N_i}^* B_{l' N_i} V_{u_j d_a} V_{u_j d_b}^* \left[F_{Box}(\lambda_{N_i}, \lambda_{u_j}) \right. \\ &\quad \left. - F_{Box}(\lambda_{N_i}, 0) - F_{Box}(0, \lambda_{u_j}) + F_{Box}(0, 0) \right] \\ &\quad + \delta_{d_a d_b} \sum_{i=1}^{n_R} B_{\tau N_i}^* B_{l' N_i} [F_{Box}(\lambda_{N_i}, 0) - F_{Box}(0, 0)]. \end{aligned} \quad (\text{A.2})$$

To our knowledge, the box function H_{Box} may not be found elsewhere in the literature. After a straightforward calculation, we obtain

$$\begin{aligned} H_{Box}(x, y) &= \frac{1}{x-y} \left[\left(4 + \frac{xy}{4} \right) \left(\frac{1}{1-x} + \frac{x^2 \ln x}{(1-x)^2} - \frac{1}{1-y} - \frac{y^2 \ln y}{(1-y)^2} \right) \right. \\ &\quad \left. - 2xy \left(\frac{1}{1-x} + \frac{x \ln x}{(1-x)^2} - \frac{1}{1-y} - \frac{y \ln y}{(1-y)^2} \right) \right]. \end{aligned} \quad (\text{A.3})$$

For the reader's convenience, we evaluate H_{Box} for some special arguments,

$$\begin{aligned} H_{Box}(x, x) &= \frac{x^3 - 15x^2 + 16x + 16}{4(1-x)^2} + \frac{-3x^3 - 4x^2 + 16x}{2(1-x)^3} \ln x, \\ H_{Box}(1, x) &= \frac{5x^2 - 39x + 16}{8(1-x)^2} - \frac{2x^3 + 16x^2}{8(1-x)^3} \ln x, \\ H_{Box}(0, x) &= \frac{4}{1-x} + \frac{4x \ln x}{(1-x)^2}, \\ H_{Box}(0, 0) &= 4, \quad H_{Box}(0, 1) = 2, \quad H_{Box}(1, 1) = \frac{7}{4}. \end{aligned} \quad (\text{A.4})$$

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Figure Captions

- Fig. 1: Feynman graphs pertinent to the semileptonic lepton-flavour-violating decays $\tau^- \rightarrow l'^- M^0$.
- Fig. 2: Branching ratios versus heavy-neutrino mass $m_N = m_{N_1} = m_{N_2}$ for the decays $\tau^- \rightarrow e^- \pi^0$ (solid line), $\tau^- \rightarrow e^- \eta$ (dashed line), $\tau^- \rightarrow e^- \eta'$ (dotted line), $\tau^- \rightarrow e^- \rho^0$ (dot-dashed line), $\tau^- \rightarrow e^- \phi$ (dot-long-dashed line), and $\tau^- \rightarrow e^- \omega$ (thick-dotted line), assuming $(s_L^{\nu_e})^2 = 0.01$, $0.02 \leq (s_L^{\nu_\tau})^2 \leq 0.05$, $\theta_P = -23^\circ$, and $\theta_V = 41.3$.
- Fig. 3: Branching ratios versus ratio m_{N_2}/m_{N_1} for the decays of Fig. 2, assuming $m_{N_1} = 1$ TeV (0.5 TeV), $(s_L^{\nu_e})^2 = 0.01$, $(s_L^{\nu_\tau})^2 = 0.05$, $\theta_P = -23^\circ$, and $\theta_V = 41.3$.
- Fig. 4: Branching ratios versus $(s_L^{\nu_\tau})^2$ for the decays of Fig. 2, assuming $m_{N_1} = m_{N_2} = 4$ TeV, $0.005 \leq (s_L^{\nu_e})^2 \leq 0.014$, $\theta_P = -23^\circ$, and $\theta_V = 41.3$.
- Fig. 5: Branching ratios versus $(s_L^{\nu_e})^2$ for the decays of Fig. 2, assuming $m_{N_1} = m_{N_2} = 4$ TeV, $0.01 \leq (s_L^{\nu_\tau})^2 \leq 0.04$, $\theta_P = -23^\circ$, and $\theta_V = 41.3$.
- Fig. 6: $B(\tau^- \rightarrow e^- \eta/\eta')$ versus $m_N = m_{N_1} = m_{N_2}$, assuming $(s_L^{\nu_e})^2 = 0.01$ and $(s_L^{\nu_\tau})^2 = 0.04$: $B(\tau^- \rightarrow e^- \eta)$ (solid line) and $B(\tau^- \rightarrow e^- \eta')$ (dashed line) for the unmixed case $\theta_p = 0$; $B(\tau^- \rightarrow e^- \eta)$ (dotted line) and $B(\tau^- \rightarrow e^- \eta')$ (dot-dashed line) for the mixed case with $\theta_p = -10^\circ$.

Table I: Quark content of the pseudoscalar meson states $|M\rangle$ and field operators $M(x)$.

$ M\rangle$	quark content of $ M\rangle$	quark content of $M(x)$
$ K^+\rangle$	$u\bar{s} \sim b_u^\dagger d_s^\dagger$	$s\bar{u}$
$ K^0\rangle$	$d\bar{s}$	$s\bar{d}$
$ \pi^+\rangle$	$-u\bar{d}$	$-d\bar{u}$
$ \pi^0\rangle$	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$ \pi^-\rangle$	$d\bar{u}$	$u\bar{d}$
$ K^-\rangle$	$s\bar{u}$	$u\bar{s}$
$ \bar{K}^0\rangle$	$-s\bar{d}$	$-d\bar{s}$
$ \eta_8\rangle$	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$
$ \eta_1\rangle$	$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$	$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
$ \eta\rangle$	$\cos\theta_P \eta_8\rangle - \sin\theta_P \eta_1\rangle$	$\cos\theta_P\eta_8(x) - \sin\theta_P\eta_1(x)$
$ \eta'\rangle$	$\sin\theta_P \eta_8\rangle + \cos\theta_P \eta_1\rangle$	$\sin\theta_P\eta_8(x) + \cos\theta_P\eta_1(x)$